D.A.V. INSTITUTIONS, CHHATTISGARH

PRACTICE PAPER-3 : 2023-24

CLASS – XII

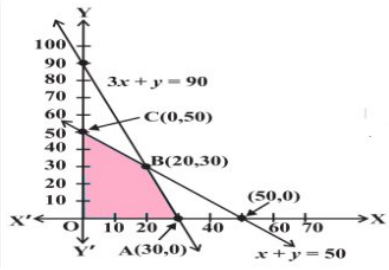
SUBJECT- MATHEMATICS (041)

Time: 3 Hrs. **Maximum Marks: 80**

General Instructions:

1. **All questions are compulsory.**
2. **The question paper has five sections. Section–A, Section-B, Section-C, Section-D and Section–E. There are 38 questions in the question paper.**
3. **Section–A has 18 MCQ questions and 2 Assertion- Reason based question of 1 marks each. Section–B has 5 Very Short Answer (VSA) type questions of 2 marks each, Section-C has 6 Short Answer (SA) type questions of 3 marks each, Section–D has 4 Long Answer (LA) type questions of 5 marks each and Section–E has 3 case based questions of 4 marks each.**
4. **There is no overall choice. However internal choice have been provided in some questions. Attempt only one of the alternatives in such questions.**
5. **Wherever necessary, neat and properly labelled diagram should be drawn.**

	SECTION-A **(Multiple Choice Questions)** **(Each question carries 1 mark)**
Q1	If A is square matrix such that $A^2 = I$, then $(A-I)^3+(A+I)^2-7A$ is equal to (a) -A (b) I-A (c) I+A (d) 3A
Q2	If $A=\begin{bmatrix} 5 & x \\ y & 0 \end{bmatrix}$ and $A=A'$,then (a) x = 0,y = 5 (b) x+y = 5 (c) x = y (d) none of these
Q3	The value of the expression $\left\|\vec{a} \times \vec{b}\right\|^2 + \left(\vec{a}.\vec{b}\right)^2$ is (a) $\vec{a}.\vec{b}$ (b) $\|\vec{a}\|.\|\vec{b}\|$ (c) $\|\vec{a}\|^2 \left\|\vec{b}\right\|^2$ (d) $(\vec{a}.\overrightarrow{b})$
Q4	If the function f defined by $f(x) = \begin{cases} \frac{x^2-9}{x-3}, & x \neq 3 \\ k, & x = 3 \end{cases}$ is continuous at x=3, then the value of K is (a) 6 (b) 3 (c) -6 (d) 3
Q5	If $f'(x) = x^2e^{x^3}$,then $f(x)$ is (a) $\frac{1}{3}e^{x^3}$+C (b) $\frac{1}{3}e^{x^4}$+C (c) $\frac{1}{2}e^{x^3}$+C (d) $\frac{1}{2}e^{x^2}$+C

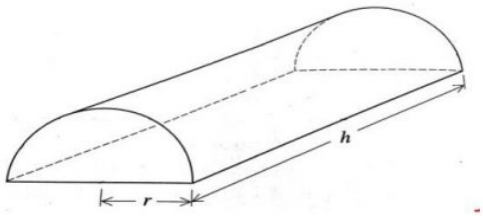
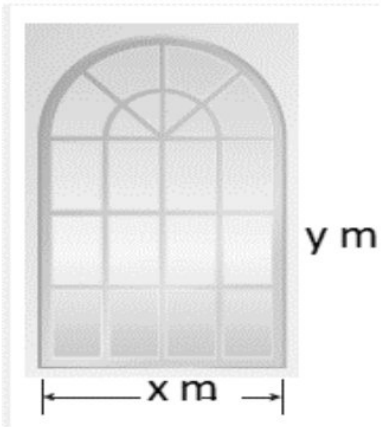
Q6	The sum of the order and degree of the following differential equation $\frac{d}{dx}\left\{\left(\frac{dy}{dx}\right)^3\right\} = 0$,is (a) 5 (b) 4 (c) 3 (d) 2
Q7	Corner points of the feasible region for an LPP are (0, 3), (1,1) and (3,0).Let Z = px + qy, where p,q>0, be the objective function. The condition on p and q so that the minimum of Z occurs at (3,0) and (1,1) is (a) p = q (b) $p = \frac{q}{2}$ (c) p = 3q (d) p=q
Q8	The value of μ such that the vectors $\vec{a} = 2\hat{\imath} + \mu\hat{\jmath} + \hat{k}$ and $\overrightarrow{b} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ are orthogonal is (a) 0 (b) 1 (c) $\frac{3}{2}$ (d) $-\frac{5}{2}$
Q9	The value of $\int_1^{\sqrt{3}} \frac{dx}{1+x^2}$ *is* (a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{12}$
Q10	If A is a square matrix of order n ,then $\lvert adj(A)\rvert$ = (a) $\lvert A\rvert$ (b) $\lvert A\rvert^{n-1}$ (c) $\lvert A\rvert^{n}$ (d) $\lvert A\rvert^{n-2}$
Q11	The corner points of the shaded bounded feasible region of an LPP are (0,0),(30,0),(20,30) and (0,50) as shown in the figure .  The maximum value of the objective function Z = 4x+y is (a) 120 (b) 130 (c) 140 (d) 150
Q12	If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$,then x is equal to (a) 6 (b) ± 6 (c) -6 (d) 0

Q13	If A is a square matrix of order 3, such that A(adjA) = 10 I , then $\|adj\ A\|$ is equal to (a) 1 (b) 10 (c) 100 (d) 101
Q14	Let A and B be two events . If P(A)=0.2,P(B)=0.4 ,P(AUB)=0.6 then $P\left(\frac{A}{B}\right)$ is equal to (a) 1 (b) 0 (c) 0.2 (d) 0.4
Q15	The integrated factor of the differential equation: $(1+x^2)\frac{dy}{dx} + y = e^{tan^{-1}x}$ is (a) $\frac{1}{e^{tan^{-1}x}}$ (b) $2\,e^{tan^{-1}x}$ (c) $3e^{tan^{-1}x}$ (d) $e^{tan^{-1}x}$
Q16	If $y = 5e^{7x} + 6e^{-7x}$,show that $\frac{d^2y}{dx^2}$ is equal to (a) 7y (b) $6y$ (c) 49y (d) 36y
Q17	The projection of the vector 2i+3j+2k on the vector i+2j+k is (a) $\frac{5\sqrt{6}}{3}$ $(b)\ \frac{5}{6}$ $(c)\ \frac{6}{5}$ $(d)\frac{\sqrt{6}}{19}$
Q18	If the direction cosines of a line are k,k,k then (a) k > 0 (b) 0 < k< 1 (c) k = 1 (d) k = $\frac{1}{\sqrt{3}}$ or k = - $\frac{1}{\sqrt{3}}$
	ASSERTION- REASON BASED QUESTIONS
	In the following questions, a statement of assertion (A) is followed by a statement of Reason (R).Choose the correct out of the following choices. (a) Both A and R are true and R is the correct explanation of A (b) Both A and R are true and R is not the correct explanation of A (c) A is true but R is false. (d) A is true but R is false.
Q19	**Assertion(A)**: The value of $cos\left(\frac{\pi}{2} + sin^{-1}\left(-\frac{1}{2}\right)\right) = \frac{1}{2}$ **Reason(R)** : $sin^{-1}(-\theta) = -sin^{-1}\theta$
Q20	**Assertion(A)**: $\frac{x+2}{-2} = \frac{y-1}{3} = \frac{z-2}{1}$ and $\frac{x-3}{-3} = \frac{y}{-2} = \frac{z+1}{2}$ are coplaner.

	Reason (R) : Let line l_1 passes through the point (x_1, y_1, z_1) and parallel to the vector whose direction ratios are $a_1, b_1, and c_1$,: Let line l_2 passes through the point (x_2, y_2, z_2) and parallel to the vector whose direction ratios are $a_2, b_2, and c_2$,. Then both lines $l_1\ and l_2$ are coplaner if and only if $\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$ =0
	SECTION -B **This section comprises of very short answer type-question (VSA) of 2 marks each**
Q21	Find the value of $\sin^{-1}[\cos(\frac{33\pi}{5})]$ OR Let y = R- $\{3\}$ and B = R- $\{1\}$. Consider the function f:A→B defined by f(x)=$\left(\frac{x-2}{x-3}\right)$. Is f is one-one and onto ? Justify your Answer.
Q22	An edge of a variable cube is increasing at the rate of 5cm per second. How fast is the volume increasing when the side is 15 cm.
Q23	Find the vector of magnitude 6,which is perpendicular to both the vectors $2\hat{\imath} - \hat{\jmath} + 2\hat{k}\ and\ 4\ \hat{\imath} - \hat{\jmath} + 3\hat{k}$ OR Find the equation of a line in vector and cartesian form which passes through the point (1,2,3) and is parallel to the vector$3\hat{\imath} + 2\hat{\jmath} - 2\hat{k}$
Q24	If $x\sin(a+y) + sina\ cos(a+y) = 0, then\ prove\ that\ \frac{dy}{dx} = \frac{sin^2(a+y)}{sina}$
Q25	If $\vec{a}$+$\vec{b}$+$\vec{c}$ =$\vec{0}$ and $\lvert\vec{a}\rvert$ =3, $\lvert\vec{b}\rvert$ =5 and $\lvert\vec{c}\rvert$ =7 then what is the angle between $\vec{a}$ and $\vec{b}$.
	SECTION C **(This section comprises of short type questions (SA) of 3 marks each)**
Q26	Find: $\int \frac{dx}{\sqrt{5-4x-2x^2}}$

Q27	Probabilities of solving specific problem independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively . If both try to solve the problem independently .Find the probability that (i) the problem is solved (ii) exactly one of them solves the problem. OR From a lot of 30 bulbs which include 6 defectives ,a sample of 4 bulbs is drawn at random with replacement. Find the probability distribution of the number of defective bulbs.
Q28	Evaluate : $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$ OR Evaluate: $\int_{-5}^{5} \lvert x+2 \rvert \, dx$
Q29	Solve the differential equation: $(1-y^2)(1+\log x)\,dx + 2xy\,dy = 0$ OR Solve the differential equation $x\,dy - y\,dx = \sqrt{x^2+y^2}\,dx$
Q30	Solve the following Linear Programming Problem graphically: Maximize: $Z = 100x + 120y$ Subject to : $5x + 8y \leq 200, 5x + 4y \leq 120, x, y \geq 0$
Q31	Evaluate: $\int \frac{x^2}{(x^2+4)(x^2+9)} dx$
	SECTION D **This section comprises of long answer -type question (LA) of 5 marks each)**
Q32	Make a rough sketch of the region $\{(x, y): 0 \leq y \leq x^2 + 1, 0 \leq y \leq x + 1, 0 \leq x \leq 2\}$ and find the area of the region using integration.
Q33	Let $A = \{1,2,3, \ldots, 9\}$ and R be the relation in $A \times A$ defined by (a, b) R (c, d) if a+ d = b+ c, for $(a, b), (c, d)$ in $A \times A$. Prove that R is an equivalence relation and also obtain the equivalence class $[(2,5)]$.

	OR Consider f:$R_+ \to [-9,\infty)$ given by f(x) =$5x^2$ +6x-9. Prove that f is bijective.
Q34	An insect is crawling along the line $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$ and another insect is crawling along the line $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$. At what points on the lines should they reach so that the distance between them is the shortest ? Find the shortest possible distance between them. OR The equation of motion of a rocket are: X = 2t, y = -4t, z = 4t ,where the time t is given in seconds, and the coordinates of a moving point in km. What is the path of the rocket ?At what distances will the rocket be from the starting point O(0,0,0) and from the following line in 10 seconds? $\vec{r} = 20\hat{\imath} - 10\,\hat{\jmath} + 40\hat{k} + \mu(10\hat{\imath} - 20\,\hat{\jmath} + 10\hat{k}$
Q35	Given two matrices A = $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and B= $\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ verify that BA=6I.Use the result to solve the system x - y = 3, 2x + 3y + 4z = 17, y + 2z =7.
	SECTION E **This section comprises of 3 case -study /passage -based questions of 4 marks each with two sub-parts.First two case study questions have three sub-parts(i),(ii),(iii) of marks 1,1,2 respectively. The third case study question has two sub-parts of 2 marks each.**
Q36	**Case – Study 1**: Read the following passage and answer the questions given below: Some young entrepreneurs started an industry “Young achievers” for casting metal into various shapes. They put up an advertisement online stating the same and expecting order to cast method for toys, sculptures, decorative pieces and more.

	A group of friends wanted to make innovative toys and hence contacted the "Young achievers" to order them to cast metal into solid half cylinders with a rectangular base and semi circular ends.  (i) If r,hand V are radius, length and volume respectively casted half cylinder, then find the total surface area function S of the casted half cylinder in terms of V and r. (ii)) For the given volume V, Find the condition for the total surface area S to be minimum. (iii) Use second derivative test to prove that Surface area is minimum for given volume. OR (iii) Find the ratio h: 2r for S to be minimum.
Q37	Dr. Rohan residing in Delhi went to see an apartment of 3BHK in Noida. The window of the house in the form of a rectangle surrounded by a semicircular opening having a perimeter of the window 10 m as shown in the figure  (i) If x and y represents the length and breadth of the rectangular region, then what is the relation between the variables.

	(ii) Dr. Rohan is interested in maximize the area of the whole window.For this to happen what should be value of x? OR (ii) For maximum value of area ,find the breadth of the rectangular part of the window. (iii) Find the maximum area of window.
Q38	Mahindra Tractors is India's leading farm equipment manufacturer. It is the largest tractor selling factory in the world. This factory has two machine A and B . Past record shows that machine A produced 60% and machine B produced 40% of the output(tractors). Further 2% of the tractors produced by machine A and 1% produced by machine B were defective. All the tractors are put into one big store hall and one tractor is chosen at random.  (i) Find the total probability of chosen tractor (at random) is defective. (ii) If in random choosing, chosen tractor is defective ,then find the probability that the chosen tractor is produced by machine 'A'